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SUPER RADIAL SYMMETRIC n-SIGRAPHS

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Abstract

An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ $(S_n = (G, \mu))$, where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ $(\mu : V \to H_n)$ is a function. In this paper, we introduced a new notion super radial symmetric n-sigraph of a symmetric n-sigraph and its properties are obtained. Also, we obtained the structural characterization of super radial symmetric n-signed graphs.

1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is refer to [1]. We consider only finite, simple graphs free from self-loops.

Key Words : *Symmetric n-sigraphs, Symmetric n-marked graphs, Balance, Switching, Super radial symmetric n-sigraphs, Complementation.*

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Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil \frac{n}{2} \rceil$.

A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ ($S_n = (G, \mu)$), where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ ($\mu : V \to H_n$) is a function.

In this paper by an *n-tuple/n-sigraph/n-marked graph* we always mean a symmetric n-tuple/symmetric n-sigraph/symmetric n-marked graph.

An n-tuple $(a_1, a_2, ..., a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. In an n-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an n-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the n-tuples on the edges of A.

In [8], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P.S.K.Reddy [4]).

Definition : Let $S_n = (G, \sigma)$ be an n-sigraph. Then,

(i) S_n is *identity balanced* (or *i-balanced*), if product of n-tuples on each cycle of S_n is the identity n-tuple, and

(ii) S_n is *balanced*, if every cycle in S_n contains an even number of non-identity edges.

Note: An i-balanced n-sigraph need not be balanced and conversely. The following characterization of i-balanced n-sigraphs is obtained in [8].

Theorem 1.1 (**E. Sampathkumar et al. [8]**) : An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.

In [8], the authors also have defined switching and cycle isomorphism of an n-sigraph $S_n = (G, \sigma)$ as follows: (See also [3], [5], [7], [10-20]).

Let $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$, be two n-sigraphs. Then S_n and S'_n are said to be *isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that if uv is an edge in S_n with label $(a_1, a_2, ..., a_n)$ then $\phi(u)\phi(v)$ is an edge in S'_n with label $(a_1, a_2, ..., a_n)$.

Given an n-marking μ of an n-sigraph $S_n = (G, \sigma)$, *switching* S_n with respect to μ is the operation of changing the n-tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. The n-sigraph obtained in this way is denoted by $\mathcal{S}_\mu(S_n)$ and is called the *μ-switched n-sigraph* or just *switched n-sigraph*.

Further, an n-sigraph S_n *switches* to n-sigraph S'_n (or that they are *switching equivalent* to each other), written as $S_n \sim S'_n$, whenever there exists an n-marking of S_n such that $\mathcal{S}_\mu(S_n) \cong S'_n$.

Two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ are said to be *cycle isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that the n-tuple $\sigma(C)$ of every cycle C in S_n equals to the n-tuple $\sigma(\phi(C))$ in S'_n.

We make use of the following known result (see [8]).

Theorem 1.2 (**E. Sampathkumar et al. [8]**) : Given a graph G, any two n-sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.

Let $S_n = (G, \sigma)$ be an n-sigraph. Consider the n-marking μ on vertices of S defined as follows: each vertex $v \in V$, $\mu(v)$ is the product of the n-tuples on the edges incident at v. *Complement* of S is an n-sigraph $\overline{S_n} = (\overline{G}, \sigma')$, where for any edge $e = uv \in \overline{G}$, $\sigma'(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S_n}$ as defined here is an i-balanced n-sigraph due to Theorem 1.1.

2. Super Radial n-Sigraph of an n-Sigraph

In a graph $G = (V, E)$, the distance $d(u, v)$ between a pair of vertices u and v is the length of a shortest path joining them. The eccentricity $e(u)$ of a vertex u is the distance to a vertex farthest from u. The radius $r(G)$ of G is defined by $r(G) = \min\{e(u) : u \in \Gamma\}$ and the diameter $d(G)$ of G is defined by $d(G) = \max\{e(u) : u \in \Gamma\}$. A graph for which $r(G) = d(G)$ is called a *self-centered graph* of radius $r(G)$. A vertex v is called an eccentric vertex of a vertex u if $d(u, v) = e(u)$. A vertex v of G is called an eccentric vertex of G if it is an eccentric vertex of some vertex of G. Let S_i denote the subset of vertices of G whose eccentricity is equal to i.

In [2], the authors introduced a new type of graph called super radial graph. The super-radial graph $SR(G)$ of a graph G on the same vertex set of G and two vertices u and v are adjacent in $SR(G)$ if and only if the distance between them is greater than or equal to $d(G) - r(G) + 1$. If G is disconnected, then two vertices are adjacent in $SR(G)$ if they belong to different components of G.

Motivated by the existing definition of complement of an n-sigraph, we extend the notion of super radial graphs to n-sigraphs as follows:

The *super radial n-sigraph* $SR(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ is an n-sigraph whose underlying graph is $SR(G)$ and the n-tuple of any edge uv is $SR(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n. Further, an n-sigraph $S_n = (G, \sigma)$ is called super radial n-sigraph, if $S_n \cong SR(S'_n)$ for some n-sigraph S'_n. The following result indicates the limitations of the notion $SR(S_n)$ as introduced above, since the entire class of i-unbalanced n-sigraphs is forbidden to be super radial n-sigraphs.

Theomre 2.1 : For any n-sigraph $S_n = (G, \sigma)$, its super radial n-sigraph $SR(S_n)$ is i-balanced.

Proof : Since the n-tuple of any edge uv in $SR(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n, by Theorem 1.1, $SR(S_n)$ is i-balanced. □

For any positive integer k, the k^{th} iterated super radial n-sigraph $SR(S_n)$ of S_n is defined as follows:

$$(SR)^0(S_n) = S_n,\ (SR)^k(S_n) = SR((SR)^{k-1}(S_n)).$$

Corollary 2.2 : For any n-sigraph $S_n = (G, \sigma)$ and any positive integer k, $(SR)^k(S_n)$ is i-balanced.

The following result characterize n-sigraphs which are super radial n-sigraphs.

Theorem 2.3 : An n-sigraph $S_n = (G, \sigma)$ is a super radial n-sigraph if, and only if, S_n is i-balanced n-sigraph and its underlying graph G is a super radial graph.

Proof : Suppose that S_n is i-balanced and G is a $SR(G)$. Then there exists a graph H such that $SR(H) \cong G$. Since S_n is i-balanced, by Theorem 1.1, there exists an n-marking μ of G such that each edge uv in S_n satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the n-sigraph $S'_n = (H, \sigma')$, where for any edge e in H, $\sigma'(e)$ is the n-marking of the corresponding vertex in G. Then clearly, $SR(S'_n) \cong S_n$. Hence S_n is a super radial n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a super radial n-sigraph. Then there exists an n-sigraph $S'_n = (H, \sigma')$ such that $SR(S'_n) \cong S_n$. Hence G is the $SR(G)$ of H and by Theorem 2.1, S_n is i-balanced. □

In [2], the authors characterize the graphs for which $SR(G) \cong \overline{G}$.

Theorem 2.4 : Let $G = (V, E)$ be a graph of order n. Then $SR(G) \cong \overline{G}$ if, and only if, G is a graph with $d(G) = r(G) + 1$ or G is disconnected in which each component is complete.

In view of the above result, we have the following result that characterizes the family of n-sigraphs satisfies $SR(S_n) \sim \overline{S_n}$.

Theorem 2.5 : For any n-sigraph $S_n = (G, \sigma)$, $SR(S_n) \sim \overline{S_n}$ if, and only if, G is a graph with $d(G) = r(G) + 1$ or G is disconnected in which each component is complete.

Proof : Suppose that $SR(S_n) \sim \overline{S_n}$. Then clearly, $SR(G) \cong \bar{(}G)$. Hence by Theorem 2.4, G is a graph with $d(G) = r(G) + 1$ or G is disconnected in which each component is complete.

Conversely, suppose that S_n is an n-sigraph whose underlying graph G is a graph with $d(G) = r(G) + 1$ or G is disconnected in which each component is complete. Then by Theorem 2.4, $SR(G) \cong \bar{(}G)$. Since for any n-sigraph S_n, both $SR(S_n)$ and $\bar{(}S_n)$ are i-balanced, the result follows by Theorem 1.2. □

Let F_{11} and F_{22} denote the set of all connected graphs G for which $r(G) = d(G) = 1$ and $r(G) = 2$, $d(G) = 3$ respectively.

The following result characterizes the n-sigraphs which are isomorphic to super radial n-sigraphs. In case of graphs the following result is due to Kathiresan et al. [2].

Theorem 2.6 : For any graph $G = (V, E)$, $SR(G) \cong G$ if, and only if, either $G \in F_{11}$ or $G \in F_{22}$ with $G \cong \overline{G}$.

Theorem 2.7 : For any n-sigraph $S_n = (G, \sigma)$, $S_n \sim SR(S_n)$ if, and only if, S_n is i-balanced and the underlying graph G belongs to either F_{11} or F_{22} with Γ is self-complementary.

Proof : Suppose $SR(S_n) \sim S_n$. This implies, $SR(G) \cong G$ and hence by Theorem 2.6, we see that the graph G satisfies the conditions in Theorem 2.6. Now, if S_n is any n-sigraph with underlying graph G belongs to either F_{11} or F_{22} with G is self-complementary, Theorem 2.1 implies that $SR(S_n)$ is i-balanced and hence if S_n is i-unbalanced and its super radial n-sigraph $SR(S_n)$ being i-balanced can not be switching equivalent to S_n in accordance with Theorem 1.2. Therefore, S_n must be i-balanced.

Conversely, suppose that S_n is i-balanced n-sigraph with the underlying graph G belongs to either F_{11} or F_{22} with G is self-complementary. Then, since $SR(S_n)$ is i-balanced as per Theorem 2.1 and since $SR(G) \cong G$ by Theorem 2.6, the result follows from Theorem 1.2 again. □

3. Complementation

In this section, we investigate the notion of complementation of a graph whose edges have signs (a *sigraph*) in the more general context of graphs with multiple signs on their edges. We look at two kinds of complementation: complementing some or all of the signs, and reversing the order of the signs on each edge.

For any $m \in H_n$, the *m-complement* of $a = (a_1, a_2, ..., a_n)$ is: $a^m = am$. For any $M \subseteq H_n$, and $m \in H_n$, the *m-complement* of M is $M^m = \{a^m : a \in M\}$.

For any $m \in H_n$, the *m-complement* of an n-sigraph $S_n = (G, \sigma)$, written (S_n^m), is the same graph but with each edge label $a = (a_1, a_2, ..., a_n)$ replaced by a^m.

For an n-sigraph $S_n = (G, \sigma)$, the $SR(S_n)$ is i-balanced. We now examine, the condition under which m-complement of $\mathcal{SR}(S_n)$ is i-balanced, where for any $m \in H_n$.

Theorem 3.1 : Let $S_n = (G, \sigma)$ be an n-sigraph. Then, for any $m \in H_n$, if $SR(G)$ is bipartite then $(SR(S_n))^m$ is i-balanced.

Proof : Since, by Theorem 2.1, $SR(S_n)$ is i-balanced, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $SR(S_n)$ whose k^{th} co-ordinate are $-$ is even. Also, since $SR(G)$ is bipartite, all cycles have even length; thus, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $SR(S_n)$ whose k^{th} co-ordinate are $+$ is also even. This implies that the same thing is true in any m-complement, where for any $m, \in H_n$. Hence $(SR(S_n))^t$ is i-balanced. □

4. Conclusion

We have introduced a new notion for n-signed graphs called super radial n-sigraph of an n-signed graph. We have proved some results and presented the structural characterization of super radial n-signed graph. There is no structural characterization of super radial graph, but we have obtained the structural characterization of super radial n-signed graph.

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